Vivekananda College

Question Bank

Mathematics Honours

Mathematics Paper – 1

Set-1

1. Let A, B, C, D be non empty sets. Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

2. Prove that a finite semigroup is a group if both the cancelation laws hold in the semigroup.

3.A relation ρ on the set of integers Z is defined by $\rho = \{(a, b) \in Z \times Z : |a - b| \leq 7\}$. Is the relation reflexive, symmetric and transitive.

4.(a) State Fermat's theorem. Use it to prove that $n^{12} - 1$ is divisible by 7 if $gcd(n, 7) = 1$.

(b) If p be a prime greater than 2, prove that $1^p + 2^p + ... + (p - 1)^p \equiv 0(mod p)$

5.(a) Solve the simultaneous congruences $x \equiv 36(mod 41), x \equiv 5(mod 17)$.

(b) If $gcd(a, b) = 1$ show that $gcd(a + b, a^2 - ab + b^2) = 1$ or 3.

6.(a) If $G = \{1, x, y\}$ is a multiplicative group then show that $x^3 = y^3 = 1$.

(b) Let a be an element of order 16 in a multiplicative group. Find the order of a^3.

7. If $AB = B$ and $BA = A$, show that the matrices A and B are both idempotent. Prove that $adj(A^t) = (adj A)^t$.

8. A is a real orthogonal matrix and $I + A$ is non-singular, I being the identity matrix. Prove that the matrix $(I + A)^{-1}(I - A)$ is a skew-symmetric.

9. Deduce the matrix A to the fully reduced normal form and find non-singular matrices P and Q such that PAQ is the fully reduced normal form where $A = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 4 & 6 \\ 3 & 0 & 7 & 9 \end{pmatrix}$

10. Find a real orthogonal matrix of order 3 having the elements $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ as the elements of a row .

11. Prove that

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-a)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c) \text{ where } 2s = a + b + c.$$

12. Prove that $\begin{vmatrix} x^3 & x^2 & x & 1 \\ \alpha^3 & \alpha^2 & \alpha & 1 \\ \beta^3 & \beta^2 & \beta & 1 \\ \gamma^3 & \gamma^2 & \gamma & 1 \end{vmatrix} = -(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(x-\alpha)(x-\beta)(x-\gamma).$

Hence deduce that $\begin{vmatrix} \alpha^3 & \alpha^2 & 1 \\ \beta^3 & \beta^2 & 1 \\ \gamma^3 & \gamma^2 & 1 \end{vmatrix} = -(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha\beta+\beta\gamma+\gamma\alpha).$

13. Find all real λ for which the rank of matrix A is 2 where $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 3 & \lambda \\ 1 & 1 & 6 & \lambda+1 \end{pmatrix}$.

14. Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$

15. If A be a real non-singular matrix prove that the matrices $A$and A^{-1} have the same set of eigenvectors and $I_n + A$ and $I_n - A$ are both non-singular matrix when A is a real skew-symmetric matrix of order n.

16. Prove that $a+b$ and $ab-h^2$ obtained from $ax^2+2hxy+by^2+2gx+2fy+c=0$ remain invariants under the transformation of rotation.

17. Find the angle through which the axes must be turned so that the equation $lx-my+n=0, (m \neq 0)$ may be reduced to the form $ay+b=0$. Find a and b.

18. Reduce the equation $3x^2-5xy+6y^2+11x-17y+13=0$ to the canonical form and state the nature of the conic.

19. Determine the values of h and g so that the equation $x^2-2hxy+4y^2+2gx-12y+9=0$ may represent (i) a conic having no centre.(ii) a conic having infinitely many centres. In case (ii) find the type of the conic.

20. Find the equation of the lines $x^2+8\sqrt{2}xy+5y^2=0$ referred to the bisectors of the angels between them as axes of coordinates.

21.(i) Prove that every infinite bounded set of real numbers has at least one accumulation point.

(ii) Prove that the derived set of a subset of real numbers is a closed set.

(iii) Prove or disprove: Union of finite number of open sets of real numbers is an open set.

22.(i) Show that a monotonic increasing sequence of real numbers is convergent if the sequence be bounded above. Hence prove that the sequence $\{x_n\}_n$, where $\{x_n\}_n = 1+\frac{1}{2^2}+\frac{1}{3^2}+...+\frac{1}{n^2}$ is convergent.

(ii) If $f: R \to R$ be continuous on R. Prove that the set $S = \{x \in R : f(x) > 0\}$ is an open set in R.

23. (i) Show that every Cauchy sequence in R is bounded.

(ii) Prove that the set of sub sequential limits of a sequence $\{a_n\}_n$ is a closed set.

(iii) If a function $f : [0, 1] \to R$ is defined by $f(x)$,

$$\text{where } f(x) = \begin{cases} 1 \text{ when x is rational} \\ 0 \text{ when x is irrartional} \end{cases}$$

then find the point of discontinuity of f. Determine the nature of discontinuities.

24. (i) Assuming Q is countable, show that $Q \times Q$ is countable.

(ii) With proper justification find a dense subset of R.

(iii) Determine all sub sequential limits of $\{\sin \frac{n\pi}{3}\}_n$.

(iv) Give an example of a set in R whose complement is neither closed nor open.

(v) Show that $\{n^{\frac{1}{n}}\}_n$ converges to 1.

Set-2

1.(a) State Bolzano-Weierstrass theorem and verify it for the set $S = \{1 + \frac{1}{n} : n \in N\}$.

(b) Show that for every real x there is a positive integer n such that $n > x$.

(c)State LUB axiom of real numbers. Use it to prove that every nonempty bounded below set of real numbers has GLB.

2.(a) Show that R has only two subsets which are both closed and open.

(b) When is a subset S of R said to be denumerable? Prove that the set of all positive rational numbers is denumerable.

(c)Define 'open set' in R. If $f : R \to R$ is continuous on R and the set S, given by $S = (x \in R : f(x) > 0)$ be a proper subset of R, then prove that S is an open set of R.

3.(a) Show that 0 is the only limit point of $\{\frac{1}{\sqrt{p}}$: p is prime$\}$.

(b) Let $f : N \to R$ be a sequence defined by

$$f(n) = \begin{cases} n!, & n = 1, 4, 7, 10, ... \\ \frac{1}{2+n}, & n = 2, 5, 8, 11, ... \\ \frac{n}{n+1}, & n = 3, 6, 9, 12, ... \end{cases}$$

Find monotone subsequences of f and determine their limits, if exist finitely. Also verify the convergence of the sequence f.

(c) Prove or disprove: $\frac{\sin x}{x}, x > 0$ is not uniformly continuous.

(d) Prove or disprove: $\lim_{x \to 0} \frac{1}{e^{\frac{1}{x}} + 1}$ exists.

4.(a)If $\{x_n\}_n$ be a cauchy sequence in R prove that $\{\cos x_n\}_n$ is also a cauchy sequence.

(b)Prove or disprove: A sequence is everywhere continuous.

(c)If $f:[a,b]\to R$ is continuous at $c\in(a,b)$ and f(c)>0 show that there exist a neighborhood N of c and a $\lambda>0$ such that $0<f(x)<\lambda$ for all $x\in N$. What happens if $f(c)=0$?

5.(a)State Intermediate value theorem.

Let the functions $f:[a,b]\to R$ and $g:[a,b]\to R$ be continuous on [a,b] and $f(a)<g(a)$, $f(b)>g(b)$. Show that there exists a point c in (a,b) such that f(c)=g(c).

(b)Prove that a real valued function satisfying Lipschitz's condition on an interval I, is uniformly continuous there.

(c) A function $f:R\to R$ satisfies the condition $f(x+y)=f(x)f(y)$ for all $x,y\in R$. If f is continuous at x=0, prove that f is continuous on R.

6.(a)Find $\overline{\lim} a_n$ and $\underline{\lim} a_n$ if $a_n=(2\cos\frac{n\pi}{2})^{(-1)^{n+1}}, n\in N$.

(b)Discuss the convergence of the sequence $\{S_n\}_{n=1}^{\infty}$, where $S_{n+1}=\sqrt{\frac{ab^2+S_n^2}{a+1}}, b>a$ for all $n\geq 1$ and $S_1=1>0$.

7.(a)Prove that $\sup\{r\in Q: r<b\}=b$, for each $b\in R$.

(b) For each positive integer n, letI_n be a closed interval and suppose
(i) $I_n\supset I_{n+1}\quad (n=1,2.......)$.
(ii)$d_n=$ length of $I_n\to 0$ as $n\to\infty$
Prove that $\cap_{n=1}^{\infty}I_n$ contains precisely one point.
What conclusion is drawn from the following facts: $\cap_{n=1}^{\infty}(-\frac{1}{n},\frac{1}{n})=\{0\}$ and $\cap_{n=1}^{\infty}(0,\frac{1}{n})=\phi$?

(c)Find the derived set of the set $\{1,\frac{1}{2},\frac{1}{3},...,\frac{1}{n},....\}$ and state with reason whether this set is closed.

8. (a)Evaluate: $\int_0^{\frac{\pi}{2}}\frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$

(b)Show that $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{n^2+2.1^2}+\frac{n}{n^2+2.2^2}+...+\frac{n}{n^2+2.(n-1)^2}\right]=\frac{1}{\sqrt{2}}\tan\sqrt{2}$.

(c)If $I_n=\int x\sin^n x dx$, prove that $I_n=-\frac{x\sin^{n-1}x\cos x}{n}+\frac{\sin^n x}{n^2}+\frac{n-1}{n}I_{n-2}(n>1)$.
Hence prove that $\int_0^{\frac{\pi}{2}}x\sin^5 x dx=\frac{149}{225}$

(d)Evaluate: $\int_0^1\frac{\log(1+x)}{1+x^2}dx$.

9. Define skew symmetric determinant. Prove that every skew symmetric determinant of order 4 is perfect square.

10. If $\Delta=\begin{vmatrix} h & a & 0\\ \frac{1}{h} & \frac{1}{b} & \frac{1}{c}\\ 0 & c & f\end{vmatrix}$ and $\Delta^{'}=\begin{vmatrix}\left(\frac{1}{bc}-\frac{1}{f^2}\right) & -\frac{1}{ch} & \frac{1}{fh}\\ af & -fh & ch\\ \frac{1}{fh} & -\frac{1}{af} & \left(\frac{1}{ab}-\frac{1}{h^2}\right)\end{vmatrix}$, then show that $\frac{\Delta^{'}}{\Delta}=-\frac{1}{acfh}$

11(a). Let A be asquare matrix of order n. Prove that $A.adj(A)=adj(A).A=det(A).I_n$ 3 (b). If A, B are two matrices such that $AB = \Theta$, can you conclude that eithe A or B is zero matrix? Justify your answer where Θ is the null matrix. 2

12. Let A be a matrix such that $(I + A)$ is non singular. Show that A is skew symmetric if $B = (I - A)(I + A)^{-1}$ is orthogonal.

13. Let $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be a basis set of the vector space V over the field F. If a non zero vector $\beta \in V$ can be expressed as $\beta = c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 + c_4\alpha_4$ $(c_1, c_2, c_3, c_4 \in F$ and $c_3 \neq 0)$ then prove that $\{\alpha_, \alpha_2, \beta, \alpha_4\}$ is a basis of V.

14. Prove that the set $S = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times 2}(R) : a + b = 0\}$ is a subspace of the vector space $M_{2\times 2}(R)$. Also find a basis of the set S and $dimS$.

15. Prove that a subset of linearly independent vectors of a vector space either a basis or it can be extend to a basis.

16. If α, β are two vectors in a Euclidean space, then prove that $|(\alpha, \beta)| \leq ||\alpha||||\beta||$

17. Define a positive definite quadratic form. Shoe that the quadratic form $5x^2+y^2+10z^2-4yz-10zx$ is positive definite.

18. Find all the eigen values of $A = \begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$. If A diagonalize then find a nonsingular matrix such that $P^{-1}AP$ is a diagonal matrix.

19. Define Cayley Hamilton theorem. Verify this theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 1 & 3 & -1 \end{pmatrix}$ and find its inverse.

20. Prove that $\nabla \times \nabla \times \mathbf{F} = \nabla(\nabla.\mathbf{F}) - \nabla^2\mathbf{F}$

21. Prove that $\nabla \times \frac{\mathbf{a}\times\mathbf{r}}{r^3} = -\frac{\mathbf{a}}{r^3} + \frac{3\mathbf{r}}{r^5}(\mathbf{a}.\mathbf{r})$

22. Prove that $\frac{d}{dt}\mathbf{a} \times \mathbf{b} = \frac{d\mathbf{a}}{dt} \times \mathbf{b} + \mathbf{a} \times \frac{d\mathbf{b}}{dt}$. Using this relation also prove that $\mathbf{a} \times \frac{d^2\mathbf{b}}{dt^2} - \frac{d^2\mathbf{a}}{dt^2} \times \mathbf{b} = \frac{d}{dt}\left(\mathbf{a} \times \frac{d\mathbf{b}}{dt} - \frac{\mathbf{a}}{dt} \times \mathbf{b}\right)$

23. If $\mathbf{a} = 3p^2\mathbf{i} - (\mathbf{p} + \mathbf{4})\mathbf{j} + (\mathbf{p^2} - \mathbf{2p})\mathbf{k}$ and $\mathbf{b} = \sin p\mathbf{i} + 3e^{-p}\mathbf{j} - 3\cos p\mathbf{k}$, then show that $\frac{d^2}{dp^2}(\mathbf{a} \times \mathbf{b}) = -30\mathbf{i} + 14\mathbf{j} + 20\mathbf{k}$ at $p = 0$.

24. If $\nabla.\mathbf{E} = 0$, $\nabla.\mathbf{H} = 0$, $\nabla \times \mathbf{E} = -\frac{\partial\mathbf{H}}{\partial t}$ and $\nabla \times \mathbf{H} = \frac{\partial\mathbf{E}}{\partial t}$ then show that $\nabla^2\mathbf{H} = \frac{\partial^2\mathbf{H}}{\partial t^2}$ and $\nabla^2\mathbf{E} = \frac{\partial^2\mathbf{E}}{\partial t^2}$

Set-3

1.State first principle of induction. Apply this to prove that the number of subsets of a set A with n elements is 2^n, n $\in N$.

2.(a) Find the least remainder when 2^{1000} is divided by 13.

(b) Prove that any positive integer other than 1 can be expressed in one and only one way in the form $2^{m_1}3^{m_2}5^{m_3}...p_n^{m_n}$ where p_n is the n-th prime number and m_r is a positive integer or zero and $m_n \neq 0$.

3. Find the least positive integer which leaves the remainders 5,7 and 3 when divided by 7,11 and 13.

4.(a)If n is a prime > 7, prove that $n^6 - 1$ is divisible by 504.

(b)Find $\phi(2048), \phi$ being Euler's function.

5.If x is real , prove that $i \log \frac{x-i}{x+i} = \pi - 2\tan^{-1} x, \text{if } x > 0.$
$= -\pi - 2\tan^{-1} x, \text{if } x \leq 0.$

6.If $\tan(\alpha + i\beta) = \tan\theta + i\sec\theta$ where α, β, θ are real and $0 < \theta < \pi$, show that $e^{2\beta} = \cot\frac{\theta}{2}$ and $\alpha = n\pi + \frac{\pi}{4} + \frac{\theta}{2}$.

7.If n be a positive integer , prove that $1!3!5!...(2n-1)! > (n!)^n$.

8.If a is any positive number except 1 and x,y,z are distinct rationals, which may be positive or negative , prove that $a^x(y-z) + a^y(z-x) + a^z(x-y) > 0$.

9.State Sturm's theorem and apply this to show that equation $x^3 - 2x - 5 = 0$ has positive root and two imaginary roots.

10.If $\alpha + \beta + \gamma = 1, \alpha^2 + \beta^2 + \gamma^2 = 3, \alpha^3 + \beta^3 + \gamma^3 = 7$ form the equation whose roots are $\alpha^2 - \beta\gamma, \beta^2 - \gamma\alpha, \gamma^2 - \alpha\beta$.

11.Show that the equation $(x-\alpha)^3 + (x-\beta)^3 + (x-\gamma)^3 + (x-\delta)^3 = 0$ where $\alpha, \beta, \gamma, \delta$ are positive and not all equal has only one real root.

12. Reduce the reciprocal equation $x^8 - x^6 + 2x^5 - 2x^3 + x^2 - 1 = 0$ to standard form and solve.

13. Solve by Ferrari's method the equation $x^4 - 18x^2 + 32x - 15 = 0$.

14.(a) If A, B, C are subsets of a universal set S then show that $[A \cap (B \cup C)] \cap [A' \cup (B' \cap C')] = \phi$.

(b) Prove that a relation ρ defined on a set A is an equivalence relation if and only if ρ is reflexive and $a\rho b, b\rho c$ together imply $c\rho a$ for $a, b, c \in A$.

15.(a) Let $f : A \to B$ and $g : B \to C$ be two mappings, prove that if gof is injective and f is surjective then g is injective.

(b) Let $f : A \to B$ be a mapping P, Q be non-empty subsets of A. Then show that $f(P \cap Q) = f(P) \cap f(Q)$ if f is injective.

16. Let $(G, *)$ be a semigroup and for any two elements a, b in G, each of the equations $a * x = b$ and $y * a = b$ has a solution in G. Then prove that $(G, *)$ is a group.

17.Let $(G, *)$ be a group of four elements e,a,b,c with e as the identity element. $a^{-1} = b$ then prove that $c^{-1} = c$. Give an example of a group G of four elements e,a,b,c with e as the identity element, where $c^{-1} = c$ but $a^{-1} \neq b$.

18. Let G be an abelian group. Prove that the set of all elements of finite order in G forms a subgroup of G.

19.(a) Transform the equation $2x^2 - xy + y^2 + 2x - 3y + 5 = 0$ to new axes of x and y given by the straight lines $4x + 3y + 1 = 0$ and $3x - 4y + 2 = 0$ respectively.

(b) Show that the equation of the line joining feet of the perpendiculars from the point (d,0) on the straight lines $ax^2 + 2hxy + by^2 = 0$ is $(a - b)x + 2hxy + bd = 0$

(c) If each of the equations $ax^2+2hxy+by^2+2gx+2fy+c = 0$ and $ax^2+2hxy+by^2-2gx-2fy-c = 0$ represents a pair of straight lines, show that the area of the parallelogram formed by them is $\frac{2c}{\sqrt{h^2-ab}}$

(d) A triangle has the lines $ax^2 + 2hxy + by^2 = 0$ for two of its sides and the point (p,q) for its orthocentre. Prove that the equation of the third sides is $(a + b)(px + qy) = aq^2 - 2hpq + bp^2$.

(e) Find the area of the triangle formed by the x-axis and the bisectors of the lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

(f) Prove that in general two parabolas can be drawn through the point of intersection of two conics $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and $a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$ and their axes are at right angles if $h'(a - b) = h(a' - b')$.

20.(a)If $(l_i, m_i, n_i), i = 1, 2, 3$ be the D.C's of three mutually perpendicular lines then show that $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = \pm 1$.

(b)Show that the straight lines whose D.C's are given by $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular or parallel according as $a^2(w + v) + b^2(u + w) + c^2(u + v) = 0$ or $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$

(c) A point P moves on a fixed plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the plane through P and perpendicular to OP meets the coordinate axes A,B,C. If the planes through A,B,C and parallel to the co-ordinate planes meet at Q, then show that the locus of Q is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}$.

(d) Show that the equation of the plane through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and perpendicular to the plane containing $\frac{x}{m} = \frac{y}{n} = \frac{z}{l}$ and $\frac{x}{n} = \frac{y}{l} = \frac{z}{m}$ is $(m - n)x + (n - l)y + (l - m)z = 0$.

(e) Prove that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0; x + z = 0; x + y = 0$ and $x + y + z = a$ is $\frac{2a}{\sqrt{6}}$ and these lines of shortest distances meet at the point (-a,-a,-a).

21.(a) Show that if $\overrightarrow{AB} = \overrightarrow{DC}$, then the figure ABCD is a parallelogram and prove that the diagonals bisect each other.

(b) Verify whether the three points $\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, $2\mathbf{a} - 3\mathbf{b} - 4\mathbf{c}$, $-7\mathbf{b} + 10\mathbf{c}$ are collinear or not?

(c) If $\mathbf{e}_1$, $\mathbf{e}_2$ be two unit vectors and θ be the angle between them, show that $2 \sin \frac{1}{2}\theta = |\mathbf{e}_1 - \mathbf{e}_2|$.

(d) Using vector method show that $\sin(A + B) = \sin A \cos B - \sin B \cos A$.

(e) Prove that $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ when and only when $(\mathbf{c} \times \mathbf{a}) \times \mathbf{b}$.

(f) Find the point of intersection of the three planes $\mathbf{r}.\mathbf{n}_1 = p_1$, $\mathbf{r}.\mathbf{n}_2 = p_2$ and $\mathbf{r}.\mathbf{n}_3 = p_3$, where $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$ are three non-coplanar vectors, not necessarily unit vectors.

$$MathematicsPaper - 2$$

Set-1

1. A particle is placed very near the vertex of a smooth cycloid, the axis being vertical and vertex upwards. Find where the particle runs off the curve. Prove that it falls upon the base of the cycloid at a distance $a(\frac{1}{2}\pi + \sqrt{3})$ from the centre of the base, a being the radius of the generating circle.

2. Find the loss of kinetic energy due to oblique impact of two smooth sphere.

3. Two particle of mass m and $2m$ are attached to the two ends of a uniform chain of mass m and length $2a$. The chain is hung over a smooth peg so that its length on two sides are equal and is then allowed to fall. Show that the chain leaves the peg after a time $\sqrt{\frac{4a}{g}} \log(2 + \sqrt{3})$.

4. A particle is projected under gravity at an angle α with the horizontal in a medium which produces a retardation equal to k times the velocity. It strikes the horizontal plane through the point of projection at an angle ω and the time of flight is T. Prove that $\frac{\tan\omega}{\tan\alpha} = \frac{e^{kT}-1-kT}{e^{-kT}-1+kT}$

5.If B=$\{(a, 0), a \in R\}$, B is a closed set but not an open subset of $R \times R$.

6.

$$\text{Let } f(x,y) = \begin{cases} x\sin\frac{1}{y} + \frac{x^2-y^2}{x^2+y^2} & \text{if } \; y \neq 0 \\ 0 & \text{if } \; y = 0 \end{cases}$$

Show that $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$ exists, but neither $\lim_{(x,y)\to(0,0)} f(x,y)$ nor $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$ exists.

7.If $f(x, y)$ is continuous at $(a, b)$and $f(a, b) \neq 0$ then there exists a neighborhood of (a, b) where $f(x, y)$ and $f(a, b)$ maintain the same sign.

8. Show that the function

$$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & \text{when } \; (x,y) \neq (0,0) \\ 0 & \text{when } \; x^2+y^2 = 0 \end{cases}$$

although continuous at (0,0) is not differentiable there.

9. If $H(x, y)$ be a homogeneous function of x and y of degree n having continuous partial derivatives and $u(x, y) = (x^2 + y^2)^{-\frac{n}{2}}$. Show that $\frac{\partial}{\partial x}(H\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(H\frac{\partial u}{\partial y}) = 0$

10. If z is a function of two variables x and y and $x = c\cosh u\cos v, y = c\sinh u\sin v$.(c is a real number), show that $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{1}{2}c^2(\cosh 2u - \cos 2v)(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2})$.

11. Show that $ax^2 + 2hxy + by^2$ and $Ax^2 + 2Hxy + By^2$ are independent unless $\frac{a}{A} = \frac{h}{H} = \frac{b}{B}$.

12. Using implicit function theorem, prove that the equation $y^2 - xy - y - x = 0$ determines y as a function of x in the neighborhood of (0,0) and find $\frac{dy}{dx}$ at (0,0).

13. Derive the approximate formula $\frac{\cos x}{\cos y} \simeq 1 - \frac{1}{2}(x^2 - y^2)$ for sufficiently small values of $|x|, |y|$.

14.Prove that the pedal equation of a circle with respect to a point on the circumference is $pd = r^2$, where d is the diameter of the circle.

15. Find asymptotes, if any, of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. If one of the

asymptotes passes through the origin, prove that $af^2 + bg^2 = 2fgh$.

16. Show that for the curve $y^2 = \frac{x^3}{2a-x}$, the origin is a cusp of first species.

17. Find the whole area included between the curve $y^2(a-x) = x^2(a+x)$ and its asymptote, $a > 0$.

18. Find the coordinates of the centre of gravity of the first arc of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$.

19. Find the envelope of the family of circles whose centres lie on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and which pass through its centre.

Set-2

1. Define an orthogonal trajectories. Find the orthogonal trajectories of the family of curves $y^2 - x^2 + 2xy - 2ax = 0$ (a is a parameter).

2. Find the general and singular solutions of the $16x^2 + 2p^2y - p^3x = 0$ where $p = \frac{dy}{dx}$.

3. Determine an integrating factor to make the equation $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ exact and hence solve it.

4. Solve by the method of undetermined coefficients the equation $(D^2 - 3D)y = x + e^x \sin x$, $D = \frac{d}{dx}$.

5. If u and v are two particular solutions of the equation $\frac{dy}{dx} + P(x)y = Q(x)$ where $P(x)$ and $Q(x)$ are continuous functions and if $v = uz$, show that $z = 1 + ae^{\int \frac{Q}{u} dx}$ where $a > 0$ is constant.

6. Find the differential equation associated with the primitive $y = a + be^{5x} + ce^{-7x}$; a, b, c being parameter.

7. Obtain the differential equation of all circles each of which touches the axis of x at the origin.

8. Find all the basic feasible solutions of the equations

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + 5x_2 - 2x_3 = 3$$

9. $x_1 = 3, x_2 = 1, x_3 = 0, x_4 = 3$ is a feasible solution to the set of equations

$$4x_1 + x_2 - x_3 + x_4 = 16$$
$$x_1 + 2x_2 + 3x_3 + 2x_4 = 11$$

Find all possible basic feasible solutions using the given solution.
10. Prove that the set of all convex combinations of a finite number of linearly independent vectors is a convex set.
Examine if $X = \{(x, y) : x^2 \geq 4y\}$ is a convex set.

11. Solve the following L.P.P.
Maximize $Z = 0.5x_2 - 0.1x_1$
subject to $2x_1 + 5x_2 \leq 80$
$x_1 + x_2 \leq 20$
$x_1, x_2 \geq 0$
by graphically.

12. Solve the following transportation problem by matrix minima method.

	D_1	D_2	D_3	
O_1	50	30	220	1
O_2	90	45	170	3
O_3	250	200	50	4
	4	2	2	

13. Solve the following L.P.P
Maximize $Z = 5x_1 + 3x_2$
subject to $x_1 + x_2 \leq 2$
$5x_1 + 2x_2 \leq 10$
$3x_1 + 8x_2 \leq 12$
$x_1, x_2 \geq 0$

14. Write down the dual problem of the following L.P.P
Minimize $Z = 6x_1 + 5x_2 + 4x_3 + 8x_4$
subject to $x_1 + x_2 \leq 700$
$6x_3 + x_4 \leq 900$
$x_1 + x_3 = 500$
$x_2 + x_4 \geq 1000$
$x_1, x_3, x_4 \geq 0$ and x_2 is unrestricted in sign.

15. solve the following zero sum game for two persons. Obtain the best strategies for both players and the value of the game:

	B_1	B_2	B_3	B_4	B_5
A_1	0	0	0	0	0
A_2	4	2	0	2	1
A_3	4	3	1	3	2
A_4	4	3	4	-1	2

16. Find the assignment from the following profit matrix.

	1	2	3	4
1	40	35	43	45
2	33	39	48	33
3	40	37	33	32
4	35	41	39	37

17. A particle of mass m executes S.H.M in the line joining the points A and B on a smooth table and is connected with these points by elastic string whose tension in equilibrium are each T. Show that the time of oscillation is $2\pi\sqrt{\frac{ma\acute{a}}{T(a+\acute{a})}}$, where a and $\acute{a}$ are the extensions of the string beyond their natural length.

18. Find radial and transverse components of velocity and acceleration.

19. A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Show that the time it takes in reaching a height h is

$$\frac{1}{3}\sqrt{\frac{2a}{g}}\left[\left(1+\frac{h}{a}\right)^{\frac{3}{2}}-1\right]$$

where a is the radius of the earth.

20. Let $f(x, y)$ be continuous at an interior point (a, b) of domain of definition of f and $f(a, b) \neq 0$. Show that there exists a neighbourhood of the point (a, b) in which $f(x, y)$ retains the same sign as that of $f(a, b)$.

21. Let $f(x,y)=\frac{xy}{\sqrt{x^2+y^2}},\ x^2+y^2 \neq 0$
$= 0, \qquad x^2+y^2 = 0$

Show that f is not differentiable at $(0, 0)$ though it is continuous at that point.

22. State and prove Schwarz's theorem for a function of two variables on the commutativity of the order of partial derivatives.

23.Given $f(x, y) = \frac{f(x)+f(y)}{1-f(x)f(y)}$, $f(x)f(y) \neq 1$, where x and y are independent variables and $f(t)$ is a differentiable function of t and $f(0) = 0$. Using property of Jacobian, show that $f(t) = \tan \alpha t$, α is a constant.

24. If $u = \frac{x+y}{z}$, $v = \frac{y+z}{x}$,$w = \frac{y(x+y+z)}{xz}$ show by using property of Jacobian that u, v, w are functionally related and find relation between them.

25. If $\beta = \{(a, 0) : a \in R\}$,β is a closed set but not an open subset of $R \times R$.

$MathematicsPaper - 3$

Set-1

1. Use convolution theorem to show that

$$L^{-1}\{\frac{p^2}{(p^2+4)^2}\} = \frac{t \cos 2t}{2} + \frac{\sin 2t}{4}$$

2. Solve the following equation by using Laplace Transform:

$\frac{d^2y}{dt^2} + 9y = cos2t$ when y(0)=1 and y($\frac{\pi}{2}$)= -1.

3. Prove that for $0 \leq x \leq \pi$,
$x(\pi - x) = \frac{8}{\pi}(\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + ...)$
Hence deduce that
$x = \frac{\pi}{2} - \frac{4}{\pi}(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...), 0 \leq x \leq \pi$

4. Verify Second Mean Value Theorem (Weierstrass'form) for the function h defined by $h(x) = x \sin x, x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

5. Show that the sequence of function $\{f_n\}_n$ on [0, 1] defined by $f_n(x) = \frac{nx}{1+n^3x^2}, x \in [0, 1]$ is uniformly convergent on [0, 1].

6.(i) Define extended complex plane C_∞. How do you represent C_∞ geometrically on a sphere? 1+3

(ii) Let (z)=$\frac{|z|}{\text{Re(z)}}$ if Re(z)$\neq$ 0 and
= 0 if Re(z)=0
Show that f is not continuous at z= 0. 3

(iii) Let C[0,1] denote the set of all real-valued continuous functions on [0,1]. For x,y$\in$ C[0,1], define d(x,y)=$\int_2^1$ |x(t)-y(t)|dt. Show that d is metric on C[0,1]. 3

7.(i) Let the function d:$R^2 \times R^2 \to R$ be defined as d(α, β)=$|x_1 - y_1| + |x_2 - y_2|$ for $\alpha, \beta \in R^2$ where $\alpha = (x_1, x_2), \beta = (y_1, y_2)$. show that d is metric on R^2. 5

(ii) Let (X,d) be a complete metric space and $\{F_n\}$ be any sequence of non-empty closed sets such that $F_1 \supseteq F_2 \supseteq F_3$...... in this space with $\lim \delta(F_n) = 0$, where $\delta(A)$ denotes the diameter of the set A. Prove that $F = \cap_{n=1}^{\infty} F_n$ contains exactly one point in X. 5

8.(i) Prove that in a metric space (X,d) any finite intersection of open sets in (X,d) is open in (X,d). Will the result remain valid if the word 'finite' is replaced by the word 'infinite'? Justify your answer. 5

(ii)Define analytic function. Construct an analytic function whose real part is $e^x \cos y$. 5

9. (a) Define random variable, probability distribution, statistical regularity, event space and mean of random variable.

(b) A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

(c) The probability of n independent events are p_1, p_2,....,p_n. Find the probability that at least one of the events will happen.

(d) A die is thrown 10 times in succession. Find the probability of obtaining six at least once.

(e) If X is uniformly distributed in the interval $(-1, 1)$, then find the distribution of $|X|$.

(f) The joint density function of the random variables X, Y is given by

$$f_{X,Y}(x, y) = \begin{cases} x + y & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the distribution of (a) $X + Y$ and (b) XY.

(g) Find the median of the normal $N(m, \sigma)$ distribution.

10. (a) Write a C programe to find the ${}^nC_r$.

(b) Write a C programe to find the numerical root of the equation $x^2 \sin x + 1 = 0$ by Newton Rapshon method.

(c) Write a C programe to find the numerical value of the integral $\int_0^1 e^{-x^2}(3x - 1)\ dx$ by Simpson's one third rule.

(d) Write a C programe to find the numerical value of the variance and mean of n given numbers.

11.(a) Show that the mapping $T : R^3 \to R$ is linear where $T(x, y, z) = 2x - 3y + 4z$

(b) Let $T : V(F) \to W(F)$ be a linear mapping. Then prove that the null space of T is a subspace of $V(F)$.

12. (a) Let $T : V(F) \to W(F)$ be a linear mapping, where $V(F), W(F)$ are finite dimensional and dimension of $V(F)$= dimension of $W(F)$. Then prove that T is one-one iff T is onto.

(b) Find the rank and nullity of the linear transformation $T : R^2 \to R^2$ defined by $T(x_1, x_2) = (x_1 + x_2, 2x_1 + 2x_2)$.

13. Find the rank, range, nullity and kernel of the linear transformation $T : R^4 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_4, -x_1 + 2x_3 + x_4, x_2 + x_3 + 2x_4)$.

14. Prove that the linear transformation $T : r_3(R) \to r_3(R)$ defined by $T(x, y, z) = (x + z, x - z, x)$ is invertible and hence find T^{-1}.

15. In the linear transformation $T : R^3 \to R^2$, the matrix of T
$M(T)=\begin{pmatrix} 1 & 2 & -3 \\ 4 & 2 & -1 \end{pmatrix}$. Determine the transformation relative to the bases $\{(1, 2, 1), (2, 0, 1), (0, 3, 4)\}$ and $\{(2, 1), (0, 5)\}$.

16. Explain the method of bisection for finding a real root of $f(x) = 0$.

17. Show that $f(x_0, x_1, x_2, ..., x_n) = \sum \frac{x_i}{\prod_{j=0, j\neq i}^{n}(x_i - x_j)}$.

18. Establish Newton-quadrature formula and hence obtain Trapezoidal rule(composite).

Set-2

1.
(a) From the axiomatic definition of probability prove that $P(A + B) = P(A) + P(B) - P(AB)$.

(b)State and prove Baye's theorem.

(c)7 Mathematics and 3 Physics books are placed at random on a book-shelf. Find the probability that none of the Physics books are placed consecutively.

(d)A and B play a game in which A's chance of winning is $\frac{2}{3}$. In a series of 8 games, what is the chance that A will win at least 6 games?

(e) A random variable X is normal (m, σ). Find the distribution of $Y = aX + b$, where a, b are constants and $a \neq 0$.

(f) Let X, Y be independent variables each having the density function $ae^{-ak}(0 < k < \infty)$, where a is a positive constant. Prove that $\frac{Y}{X+Y}$ is uniformly distributed over (0,1).

(g) If X, Y are independent Poisson variables with parameters μ_1, μ_2 respectively, then show that $X + Y$ is a Poisson variate with parameter $\mu_1 + \mu_2$.

(h) The random variables X, Y are connected by the linear relation $2X + 3Y + 4 = 0$. Show that $\rho(X, Y) = -1$.

(i) Prove that the correlation coefficient $\rho(X, Y)$ lies between (-1,1).

(j) State and prove law of large numbers.

2.
(a)Write a c-programe to find the transpose of a given matrix.

(b)Write a c-programe to find the numerical value of the integral $\int_a^b e^{\tan^{-1}(\sinh x)}dx$ by Simpson's $\frac{1}{3}$ rule.

(c)Write a c-programe to check whether a given positive number is prime or not.

(d)Write a c-programe to find a root of the equation $f(x) = x^2 \sin x + 1 = 0$ by Newton-Rapshon's method correct upto 4D places.

(e)Write a c-programe to find the value y from the equation $y = f(x)$ for $x = x_1, x_2, x_3, x_4, ..., x_n$ where

$$\begin{aligned} f(x) &= 2(x-1), x > 1 \\ &= 2(1-x), x < 1 \\ &= 0, \qquad x = 1 \end{aligned}$$

(f) Define for loop, switch statement and difference between $a++$ and $++a$.

(3) Derive an expression of the error involved in approximating a function by an interpolating polynomial when the values are known at $(n+1)$ distinct points. 6

(4) Establish Gauss-Seidal iteration method to solve a system of n linear equation in n unknown. Comment on the convergence of the method.

(5) (i) Prove that under linear transformation $x = \alpha + \beta t$, where $x_i = \alpha + \beta t_i,\ i = 0, 1, 2, 3, ..., n$ and $f(\alpha + \beta t) = F(t)$, $\quad f(x_0, x_1, x_2, ...x_n) = \beta^{-n} F(t_0, t_1, ..., t_n)$.

(ii) Define confluent divided difference and show that $f(x_0, x_0, x_0) = \frac{1}{2} f''(x_0)$.

(6) Find $y(4.4)$, by Euler's modified method, taking $h = 0.2$, from the differential equation $\frac{dy}{dx} = \frac{2-y^2}{5x}$, where $y(4) = 1$.

7.(a) Find the rank, range, nullity and kernel of the linear transformation $T : R^4 \to R^3$ such that

$T(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 + 3x_4, -x_1 + 2x_3 + x_4, x_2 + x_3 + 2x_4)$.

(b) Let V, W be vector spaces over a field F. If a linear mapping $T : V \to W$ is invertible, prove that T is one-one, onto and conversely.

(c) In the linear mapping $T : R^3 \to R^2$, the matrix of $T = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 2 & -1 \end{pmatrix}$. Determine the transformation relative to the bases $\{(1, 2, 1), (2, 0, 1), (0, 3, 4)\}$ and $\{(2, 1), (0, 5)\}$

8. (a) Prove that a linear transformation T on a vector space V is non-singular iff the image of a basis of V is again a basis of V under T.

(b)Prove that the linear transformation $T : V_3(R) \to V_3(R)$ which maps the basis vectors

$\{(1,2,2),(2,1,2),(2,2,1)\}$ to $\{(0,1,1),(1,0,1),(1,1,0)\}$ is one-one and onto.

(c)If $(\alpha_1,\alpha_2,\alpha_3),(\beta_1,\beta_2,\beta_3)$ be the ordered basis of the vector spaces $U_3(R), V_3(R)$ respectively and $T : U_3 \to V_3$ be a linear transformation such that $T(\alpha_1) = \beta_1+\beta_2, T(\alpha_2) = \beta_2+\beta_3, T(\alpha_3) = \beta_3$. Find the matrix of T relative to the ordered basis $(\alpha_1,\alpha_2,\alpha_3)$ of U and $(\beta_1,\beta_2,\beta_3)$ of V. Show that T is non-singular. Also find the matrix of T^{-1} relative to the same ordered bases.

9. If A_i is covariant vector show that $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ are the components of a tensor.

10. If $b^{ij}u_iu_j$ is an invariant for arbitrary covariant vector u_i then show that $b^{ij}+b^{ji}$ is a contravariant tensor of rank 2.

11. Prove that $-g^{kj}\{{}^{l}_{ij}\} - g^{lj}\{{}^{k}_{ij}\} = \frac{\partial g^{kl}}{\partial x^i}$.

12.(i)Show that the vectors $(-1,0,0,\frac{1}{c})$ is a null vector for the space whose metric is $(ds)^2 = -(dx)^2 - (dy)^2 - (dz)^2 + c^2(dt)^2$.

(ii) If A^i and B^j are orthogonal unit vectors show that $(g_{ij}g_{kl} - g_{ik}g_{jl})A^iB^lA^jB^k = 1$

13. If A^{ij} is a symmetric tensor, prove that $A^j_{i,j} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^j}(A^j_i\sqrt{g}) - \frac{1}{2}A^{jk}\frac{\partial g_{ik}}{\partial x^i}$ where $A^j_i = A^{jk}g_{ik}$

Set-3

1.Starting from the power series expression of $(1+x^2)^{-1}$,with proper justification, show that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$.

2.Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series such that $\lim_{n\to\infty}\sup|a_n|^{\frac{1}{n}} = R, 0 < R < \infty$. Prove that the given series is absolutely convergent for $|x| < \frac{1}{R}$ and divergent for $|x| > \frac{1}{R}$.

3.Show that a subset E of R is compact if every infinite subset of E has an accumulation point in E.

4.Let $E = \{r \in Q : \sqrt{2} < r < \sqrt{3}\}$. Show that E is not compact in Q, but E is closed and bounded in Q. Explain why this does not contradict the Heine-Borel theorem.

5.Show that the function f defined on [0,1] by $f(x) = x\cos\frac{\pi}{2x}, 0 < x \le 1$ and $f(0) = 0$ is continuous but not of bounded variation on [0,1].

6.Let $f : [a,b] \to R$ be a function of bounded variation on $[a,b]$ and T be the variation function of f on $[a,b]$.Then f is continuous at a point c in $[a,b]$ iff T is continuous at c.

7.Show that $f : [0,2] \to R$ defined by $f(x) = 0$ where $x = \frac{n}{n+1}, \frac{n+1}{n}, (n = 1,2,3,\ldots)$
=1, elsewhere
is integrable on $[0,2]$ and $\int_0^2 f(x)dx = 2$.

8.Let $\{f_n\}_n$ be a sequence of Riemann integrable functions on an interval $[a,b]$. If $f_n \to f$ uniformly on $[a,b]$, prove that f is Riemann integrable on $[a,b]$.

9.Let $f : [a,b] \to R,\ g : [a,b] \to R$ be both Riemann integrable on $[a,b]$. Prove that the function fg defined by $(fg)(x) = f(x)g(x)$ for all $x \in [a,b]$ in Riemann-integrable on $[a,b]$.

10.Show that the sequence of funtion $\{f_n\}_n$ defined by $f_n(x) = nx(1-x)^n, x \in [0,1]$ is not uniformly convergent.

11.Using Laplace transform solve $\frac{d^2x}{dt^2} - \frac{dx}{dt} = e^{3t}$ when $x'(0) = 0, x(1) = 0$.

12.Let $f : [-\pi,\pi] \to R$ be defined as follows :

$$f(x) = \begin{cases} \cos x & 0 \le x \le \pi \\ -\cos x & -\pi \le x < 0 \end{cases}$$

Obtain the Fourior's coefficients and the Fourior series for the function $f(x)$. Hence find the sum of the series $\frac{2}{1.3} - \frac{6}{5.7} + \frac{10}{9.11} - \ldots$

12. Show that the harmonic function satisfy the differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$

13.Define Stereographic Projection. For the point $\frac{1-i}{\sqrt{2}}$ in the complex plane $\mathbb{C}$ find the corresponding image point on the Riemann Sphere $x^2 + y^2 + (z - \frac{1}{2})^2 = \frac{1}{4}$.

14.If f be an analytic function on a region D such that $\lim f = 0$, then show that f is constant.

15.Let G be an open disc in $\mathbb{R}^2$ and u, v be real valued functions defined on G. Prove that $u + iv$ is continuous on G if and only if u, v are continuous on G.

16.Define analytic function. Apply Milne-Thompson's method to construct an analytic function whose real part is $e^x \cos y$.

17. Let X be a non empty set and consider the discrete metric'd' defined on X. Show that all convergent sequences in (X,d) are eventually constant sequence .

18.Show that, for all $x, y \in R, d(x,y) = |\tan^{-1} x - \tan^{-1} y|$ defines a metric on R, which is bounded too.

19.Prove that $d(A \cup B) \le d(A) + d(B) + d(A,B)$ where d=diameter and d(A,B)=distance between the set A and B.

20.Check whether in a metric space (X,d) the empty set ϕ and the whole space X are open set or not and show that every open sphere in a metric space (X,d) is an open set.

21.Consider the metric spaces (R, d_1) and (R, d_2) where $d_1(x,y) = |x - y|$ for all x,y in R, and d_2 is the discrete metric on R. Show that the sequence $\{\frac{1}{n}\}$ converges to 0 in (R, d_1) and the sequence $\{\frac{1}{n}\}$ does not converges to 0 in (R, d_2).

22. Evaluate $\int_0^\infty \log(x + \frac{1}{x}) \frac{dx}{1+x^2}$.

23. Discuss the convergence of $\int_0^\infty \log \sin x dx$.